FWCG 25 Open Problems

1. Sam Cohen: Consider an NP-Hard decision problem $P(S)$, where S is a collection of objects. Define the *flex* version of P as flex-$P(S) = \bigwedge_{x \in S} P(S \setminus \{x\})$, asking if P always holds given the removal of any element in an input set. For example, flex-SubsetSum asks whether a set contains a subset summing to a target value when any element is removed. Is flex-Partition NP-Hard? (Given a set of values, can the set be partitioned into two equal-sum subsets when any element is removed)?

2. Reilly Browne: Finding the minimum partition of an orthogonal polygon into histogram polygons is known to be NP-hard. Can we obtain any approximation algorithm for this?

   There is a 6-approximation for covering in the x-monotone case.

3. Linh Nguyen: Given an arrangement of n lines $l_1, l_2, \ldots, l_n$ in the plane and 2 points $p_1 \in l_i, p_2 \in l_j$. Can we compute the shortest path (with respect to Euclidean length) along the edges of the arrangement between p_1 and p_2 in sub-quadratic time? The simplest solution involves building the planar graph of the arrangement in $O(n^2)$ time and apply any shortest path algorithm. To do this in sub-quadratic time would mean bypassing building the arrangement.

   Known partial solutions: sub-quadratic achievable if the lines have a bounded number of orientations (Eppstein, Hart SODA'99), or if the lines can be partitioned into two sets S_1, S_2, where lines in set S_1 all pass through a point q_1 and lines in set S_2 all pass through a point q_2 (Hart CCCG'03). Also a near linear time constant factor approximation in the general case is known (Bose, Evans, Kirkpatrick, McAllister, Snoeyink CCCG'96)

4. Reilly Browne: Given a set of axis-parallel line segments, can we find a (2-ϵ) approximation for the maximum independent set (i.e. largest maximum disjoint set)?

   ———-Proposed During the Session———-

5. Peter Brass: Which connected object C of diameter one in 3-dimensional space maximizes $\max g \in \mathrm{conv}(C)$ of $\min p, q \in C$ of the distance of g to the line pq?

   Interpretation: C is an object we want to hold with two hands, e.g., a chair or a tripod. We hold the object at points $p, q \in C$, e.g., points on two of the legs of the tripod. The object then generates a torque around this axis, the line pq, which is proportional to the distance of the line to the center of gravity g. That point g can be anywhere in the convex hull of C. So we are asking for the object of diameter one that generates most torque when we try to lift it with two hands.

   As example, consider the object T consisting of the three edges of a tetrahedron that meet in a vertex v, and choose as center of gravity g the center of the face of the tetrahedron opposite to v. All possible lines through two points $p, q \in T$ lie in the planes of the faces adjacent to v, so the torque in this example is proportional to the distance of the face center g to one of the other faces.

   Without the connectedness assumption, I believe the four vertices of a regular tetrahedron are the worst set.

6. Alex Andoni: Fix $\epsilon > 0$. Consider A, B on unit sphere in R^d of size $n \geq 2^d$. s.t $|\langle a, b\rangle| \leq \epsilon$ for all $a \in A, b \in B$. There exists linear $d/2$-dimensional subspace L and $A' \subset A$ or $(B' \subset B)$ of size $\geq \sqrt{n}$ that is $\sqrt{\epsilon}$-close to L.

7. Hugo Akitaya: Cops and robber game as usual, now as in Ron's talk. Cop catches robber if he sees him. Define the continuous visibility cop-width of a polygon, minimum number of cops to guarantee that we can catch a robber.

   In a simple polygon, the cop-width is 1: triangulate the polygon and look at the dual tree. The same strategy as the "graph cop" in this tree also works for the polygon (a cop can see the entire triangle since it is convex). Indeed, the continuous visibility cop-width is upper-bounded by the graph cop-width in the dual of a triangulation of the polygon by the same argument. Is there a polygon for which this upper bound is not tight? Jayson answered this question by coming up with an example (a square with small square holes in a grid) where 3 cops are enough to catch the robber, but there exist a triangulation with $O(\sqrt{n})$ treewidth (graph cop-width).

   Joe: related work in SODA paper, minimum set of cops that are pairwise-visible along a chain in simple polygons. This version is similar to the invisible robber model, which relates with pathwidth similarly to how cop-width relates to treewidth.

   Another interesting question is to determine strategies to catch the robber as fast as possible.

8. Jack Spalding-Jamieson: *Obnoxious k-median problem.* Given a metric space (X, d) and a value k. Output a set $S \subseteq X$ maximizing $\sum_{x \in X} \min_{s \in S} d(x, s)$. $|S| = k$

   This problem is known to be NP-hard, but no approximation results are known.

   Is a constant factor approximation possible? Or a pseudo-approximation (e.g., a set $|S| \geq \frac{k}{2}$ approximating the best value for a set $|S'| = k$)?

9. Mayank Goswami: Diameter/Farthest Earth Movers Distance Given a metric space of n points P and some number k. Select two subsets S_1, S_2, each of cardinality k, such that $EMD(S_1, S_2)$ is maximized.

   This is known to be NP-hard in general metrics, unknown in Euclidean

   Can reduce with loss of factor 1/2 if you fix one of the subsets.

   Arie Prior work: cut version has a 1/2 approximation

   Samuel Cohen: counterexample to doing diameters greedily. Answers Jayson's question about greedy

10. Arie Tamir: Upper Envelope of a set of functions. If we are given pseudolines, the number of breakpoints is linear. 2-intersections also holds linear. 3-intersection points, there is a lower bound of $n\alpha(n)$ (See Davenport-Schintzel sequences). Now consider the minimum/max function, you are given linear functions. You can insert/delete say n lines. At each point in time, the upper envelope will have a particular complexity. What happens if you do n insertions and deletions, what is the total number of points that will appear in the process of insertions and deletions.

    There are data-structures can find this in $O(n^2 \log n)$, you can show $n \log n$ is an upper bound on total number of intersection points that will ever appear. Lowerbound is not linear, it is $n\alpha(n)$.

    Given a bunch of segments with descending ends, the complexity of the upper envelope is $n\alpha(n)$. To make it a function, we get that it is 3-intersection, hence this is tight.

11. Arie Tamir: Points on the line, consider them to be clients. Choose k servers, each client will walk to nearest server. Most k-median types are easy on the line.

    Consider the min-max and the min-sum, instead of just largest we care about p-best servers instead.

    Open: More general objective function called the convex median objective function. Given p distances, choose a set of k-servers. $d(s_i, x_k)$, distance to the closest one. For each x_k, there is a subset of n numbers for a given server, sort them from largest to smallest. Fixed coefficients chosen on the weighted sum of the elements. Input into problem is a set of coefficients for each point. Weighted k-servers.

12. Joe Mitchell: Recall a problem posed by Arie Tamir at the Fourth NYU Computational Geometry Day (March, 1987): "Given a collection of compact sets, can one decide in polynomial time whether there exists a convex body whose boundary intersects every set in the collection?"

    A partial answer is given in our paper [ADK+14]: if the input sets can possibly intersect (e.g., for a collection of possibly intersecting line segments), deciding if there is a convex transversal is NP-hard. Also, if there is a set of candidate vertices for a convex transversal polygon, and the input sets are segments (or polygons), then the problem can be solved in polynomial time (using dynamic programming).

    What can be said if no set of candidate vertices is given and the input sets are disjoint line segments?

13. Csaba Tóth: Erdős-Szekeres Maker-Breaker game. Given an integer $k \geq 7$, two players place points in the plane in turns. In round i, $i \in \mathbb{N}$, Maker places one point in the plane and Breaker places $f(i)$ points. Maker wins if the points placed so far form an empty convex k-gon, and Breaker wins if the game continues forever. We know that if $f(i)$ is a constant, then Maker wins for any $k \geq 3$ [DPP+25], and there exists a function f for which Breaker wins for any $k \geq 7$ [CL23].

    What is the smallest function f for which Breaker wins?

References

[ADK+14] Esther M Arkin, Claudia Dieckmann, Christian Knauer, Joseph SB Mitchell, Valentin Polishchuk, Lena Schlipf, and Shang Yang. Convex transversals. *Computational Geometry*, 47(2):224–239, 2014.

[CL23] David Conlon and Jeck Lim. Fixing a hole. *Discret. Comput. Geom.*, 70(4):1551–1570, 2023. doi:10.1007/S00454-023-00561-6.

[DPP+25] Aleksa Dzuklevski, Dömötör Pálvölgyi, Alexey Pokrovskiy, Csaba D. Tóth, Tomás Valla, and Lander Verlinde. Erdős-Szekeres maker-breaker games. In *Proc. 31st International Computing and Combinatorics Conference (COCOON), Part I*, pages 248–261, 2025. doi:10.1007/978-981-95-0215-8_19.